Tapping into our information processing capacity: A novel methodology for cognitive analysis using multiplicative cascades

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Think beyond the square
Think instead of

A wonderful experience, being in the flow, a stream of consciousness

Beautiful mountain vistas

Magical waterfalls

Stirring orchestral music with instruments playing at different speeds within the same temporal framework

The excitement of INTERMITTENT turbulence on your next flight across the Pacific

Human responses as singularities (abrupt changes) in an otherwise continuous stream
Also imagine

Not having to worry about what response was made, only when it occurred

Being able to ignore residual times for perception and response execution

Being able to accommodate all sorts of sequential dependencies automatically

Caring slightly less about nonstationarity in the RT data
The challenge is to determine $p(x)$ from the RT series it generates.
Cheng (2014):
General Binomial Model

$k = 1$: there are two \( (2^1) \) temporal partitions with capacity vector \( \{ p_1, p_2 \} \)

\[ k = 2: \text{there are four} \ (2^2) \ \text{temporal partitions with capacity vector} \ \{ p_1 \{ p_1, p_2 \}, p_2 \{ p_1, p_2 \} \} = \{ p_1^2, p_1 p_2, p_2 p_1, p_2^2 \} \]

A measure of the \( i \)-th temporal subinterval: \( \mu_i(\Delta t_2) = p_1^i p_2^{2-i}, \ i = 0, 2, \text{ and } \Delta t_2 = h^{-2}, h \geq m_1 + m_2 \)

One-q-th moment of \( i \)-th of \( n \) measures, \( \Delta t_n \), at stage \( k = n \) is given by \( \mu_i(\Delta t_n)^q = [p_1^i p_2^{n-i}]^q, \ q \in (-\infty, 0) \cup (0, \infty) \).

More generally, \( \chi_q(\Delta t_n) = \Sigma_{i=0}^n \mu_i(\Delta t_n)^q m_1^i m_2^{n-i} \binom{n}{i} = \Sigma_{i=0}^n [p_1^i p_2^{n-i}]^q m_1^i m_2^{n-i} \binom{n}{i} = [m_1 p_1^q + m_2 p_2^q]^n \)

When \( \chi_q(\Delta t_n) = \Delta t_n^{\tau(q)} \) and \( \Delta t_n = h^{-n} \),

\[ \tau(q) = - \frac{\log[m_1 p_1^q + m_2 p_2^q]}{\log(h)} \] This is the multifractal scalar function.
The singularity index function, \( a(q) \), is given by

\[
a(q) = \frac{\partial \tau(q)}{\partial q} = -\frac{\xi_q \log(p_1) + (1 - \xi_q) \log(p_2)}{\log(h)}
\]

where \( \xi_q = \frac{m_1 p_1^q}{m_1 p_1^q + m_2 p_2^q} \)

The multifractal spectrum function is given by:

\[
f(a) = a(q)q - \tau(q) = -\frac{\xi_q \log \left( \frac{\xi_q}{m_1} \right) + (1 - \xi_q) \log \left( \frac{1 - \xi_q}{m_2} \right)}{\log(h)}
\]

Positive \( q \) \( \rightarrow \) large scale fluctuations
Negative \( q \) \( \rightarrow \) small scale fluctuations

**BUT**

Large \( a \) \( \rightarrow \) negative \( q \) \( \rightarrow \) small scale fluctuations
Small \( a \) \( \rightarrow \) positive \( q \) \( \rightarrow \) large scale fluctuations
DOES WELL DOES THE GENERAL BINOMIAL MODEL FIT THE DATA?
Four choice task

Stimuli presented
One per second

Each of the three sessions contained 2400 trials

Multifractal scalar function

Multifractal spectrum function
WHAT ARE THE PREDICTIONS OF OTHER pdfs THAT DRIVE THE MULTIPLICATIVE CASCADE?
<table>
<thead>
<tr>
<th>Elementary Process</th>
<th>PDF</th>
<th>Multifractal spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson</td>
<td>$p(x) = \frac{e^{-\gamma} \gamma^x}{x!}$</td>
<td>$f(\alpha) = 1 - \frac{\gamma}{\log(2)} + \alpha \log_2 \left( \frac{\gamma e}{\alpha} \right)$</td>
</tr>
<tr>
<td>Gamma</td>
<td>$p(x) = \frac{\beta^\gamma x^{\gamma-1} e^{-\beta x}}{\Gamma(\gamma)}$</td>
<td>$f(\alpha) = 1 + \gamma \log_2 \left( \frac{\alpha \beta}{\gamma} \right) + \frac{\gamma - \alpha \beta}{\log(2)}$</td>
</tr>
<tr>
<td>Restricted Gamma</td>
<td>$\gamma = \kappa \beta$</td>
<td>$f(\alpha) = 1 + \kappa \beta \log_2 \left( \frac{\alpha}{\kappa} \right) + \frac{\beta (\kappa - \alpha)}{\log(2)}$</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$p(x) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left[ - \frac{(x - \mu)^2}{2\sigma^2} \right]$</td>
<td>$f(\alpha) = 1 - \frac{1}{2\log(2)} \left( \frac{\alpha - \mu}{\sigma} \right)^2$</td>
</tr>
</tbody>
</table>

$E[p(x)] = \text{value of } \alpha \text{ when } f(\alpha) = 1,$

i.e. the location of the peak of the multifractal spectrum

Var[$p(x)$] is determined by the width of the multifractal spectrum.

From: Calvert, Fisher and Mandelbrot (1997)
Restricted Gamma fits as well as the Gaussian
The restricted Gamma fits the best
The restricted Gamma fits the best
How are the Multifractal Scalar and Spectrum Functions estimated from the RT time series?
Let the RT time series be: $x(i), i = 1, N$

Compute the summed deviations about the mean: $y(k) = \sum_{i=1}^{k}[x(i) - \bar{x}], k = 1, N$

Partition $y(k)$ into $N/n$ nonoverlapping subsequences of size $n$, then fit a linear segment to each subsequence to produce $y(n,k)$

Compute the average fluctuation of the integrated series about the trend line as

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^{N} [y(k) - y(n,k)]^2}$$

The slope of the plot of $\log(F(n))$ against $\log(n)$ is an index used in Detrended Fluctuation Analysis to investigate long-range dependence in time series data: slope $= 0.5 \rightarrow$ Gaussian, $0.5 < \text{slope} < 1 \rightarrow$ persistence, $0 < \text{slope} < .5 \rightarrow$ antipersistence.

We will generalize this method to produce MultiFractal Detrended Fluctuation Analysis (Kandelhardt, 2002)
Divide the cumulative time series subsequence into $N_s \equiv \text{int}(N/s)$ nonoverlapping segments, each of size $s$

Fit the data for each segment $\nu$, $\nu = 1, N_s$, with a polynomial $\hat{y}_\nu(i)$ and compute the fluctuation function:

$$F^2(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} \{y[N - (\nu - N_s)s + i] - \hat{y}_\nu(i)\}^2$$

Average over all segments to obtain the $q$-order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(s, \nu)]^{q/2} \right\}^{1/q}$$

Let $F_q(s)$ be a power function of $s$, $F_q(s) = s^{h(q)}$, and estimate $h(q)$ from which the multifractal scalar function can be estimated as $\tau(q) = qh(q) - 1$. Then

$$\alpha = \frac{d\tau(q)}{dq}$$

and the multifractal spectrum,

$$f(\alpha) = q\alpha - \tau(q)$$

can be estimated using a Legendre transform. $\alpha$ is the $q$-order singularity strength or Hölder exponent.

I have modified the MATLAB routine in Ihlen (2012) as an R function.
How does this all work when we give people a challenging task to perform?
Four Choice Task (Kelly, Heathcote, Heath & Longstaff, 2001)

On each trial, one of four visual stimuli was illuminated and each of 12 subjects was required to depress the corresponding response button.

Conditions:
1. **Self-paced**: Respond as quickly and as accurately as you can (The usual instructions!) All but 1 observation were valid.
2. **Fast-paced Fast**: ISI = mean RT in condition 1. Average deadline = Mean ISI = 449 ms, 94% of observations were valid.
3. **Fast-paced Slow**: ISI = mean RT + 2SD as estimated from condition 1. Average deadline = Mean ISI = 578 ms, 98.5% of observations were valid.

Each block of trials lasted no more than 4 minutes to minimize fatigue. Each session contained 3120 trials, except for the trials on which there was no response before the next stimulus appeared.
Gaussian model provides the best fit for four of the 12 subjects, the restricted Gamma model providing the best fit for the other subjects.
Restricted Gamma model provides the best fit for all 12 subjects.
Restricted Gamma model provides the best fit for all 12 subjects.

Fast-Paced Slow Task
No significant difference in the \( \kappa \) parameter.

Average value = 0.6
Let Capacity be represented by the shape parameter of the restricted Gamma (number of stages in the equivalent Erlang process).

So Capacity = $\beta \kappa$

Capacity = $22.2 - 0.0757$ Mean RT

<table>
<thead>
<tr>
<th>Task</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Paced</td>
<td>4</td>
</tr>
<tr>
<td>Fast-Paced Slow</td>
<td>7</td>
</tr>
<tr>
<td>Fast-Paced Fast</td>
<td>9</td>
</tr>
</tbody>
</table>

Fast-Paced capacity estimates look MAGICAL (Miller, 1956).

Self-Paced task capacity = transient STM capacity (Grossberg, 1978; Cowan, 2001)
NOW TO FINISH WITH A LITTLE EXPERIMENT

I INVITE YOU TO READ THE TEXT ON THE NEXT SCREEN WHILE TAPPING AWAY WITH YOUR DOMINANT FOREFINGER AT A RATE THAT DOES NOT AFFECT YOUR COMPREHENSION OF THE TEXT.

PLEASE INVITE THE PERSON NEXT TO YOU TO ESTIMATE THE NUMBER OF TAPS PER SECOND.
Today you have learned some amazing things about human response time. Rather than use conventional methodology we have just analyzed the series of response times without caring at all about the properties of the stimuli, except for their fixed presentation rate (in all but the self-paced condition).

We used a new data analysis technique known as multifractal detrended fluctuation analysis to determine the precise form of the random multiplicative cascade process that produced the series of response times in two experiments. In most cases, especially in the speeded tasks, this turned out to be a restricted gamma process in which the shape parameter is proportional to the rate parameter.

When we related a derived capacity measure to the mean RT averaged over all the subjects but computed separately for each condition in Experiment 2, we obtained an exact linear relationship. From this we deduced that the capacity estimates were precisely those obtained in short-term memory tasks when people are actually asked to remember items for later recall. Isn’t it amazing how we could get the same numbers without asking subjects to remember anything?

Now that you have been tapping along for a few seconds I suggest you stop reading this and ask the person sitting next to you to tell you how many taps you produced per second (approximately).
If you extrapolate the linear fit so that it crosses the Capacity = 0 line you will notice that this mean RT is 293 msecs.

If you were paying full attention to my summary slide and not using any excess cognitive capacity to produce your taps you should have tapped around 3 times per second. This would further confirm the linear relationship I found in Experiment 2.

When people were invited to tap a key at their normal rate, the mean inter-tap interval was 297 msecs (Hansen, Ebbesen, Dalsgaard, Mora-Jensen, & Rasmussen, 2015), a value very close to that estimated from the Experiment 2 data for zero capacity responding (293 msecs).

It is interesting that this tapping rate would coincide with the regular rhythm in Western music of up to 3 Hz with possible links to delta EEG oscillations (Merchant, Grahn, Trainor, Rohrmeier & Fitch, 2015).
Just in case you are tempted to go off the rails and try a bit of THC (I dare not mention what it is), I wouldn’t if I were you. It will stuff up the multifractality of the hippocampal cells that assist performance in a discriminative learning task (blue curve in the FCT graph, Fetterhoff et al., 2015).

Dulled by the THC, the cells in CA1 and CA3 will behave a bit like a diffusion process and we couldn’t have that (peak location of the green THC curve in the FCT graph is at $h = 0.5$, a Brownian motion process, as is also the case for non-responding cells)!

However we see that for rats, efficient memorization is reflected in greater multifractality of hippocampal cell function, suggesting that the findings from our CRT task may have a strong neurophysiological basis.
Thanks for coming today,

Do you have any questions?


